ගම්පහ අධ්‍යාපන කලාපය
Gampaha Education Zone

දෙවන වාර ඇගයීම - 2025
Second Term Evaluation - 2025
இரண்டாம் தவணைப் பரீட்சை - 2025

ශ්‍රේණිය Grade **13**	විෂයය Subject **Combined Mathematics II**	කාලය Time **3 h 10min**
නම பெயர் Name		

Instructions:

- This paper consists of two parts.
  Part A (Questions 1-10) & **Part B (Questions 11-16)**
- **Part A**
  Answer **all** the questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- **Part B**
  Answer **five** questions only. Write your answers on the sheets provided.
- At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.

You are permitted to remove **only Part B** of the question paper from the examination hall.

For Examiners' Use Only.

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	**1**	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	**11**	
	12	
	13	
	14	
	15	
	16	
	Total	
	Percentage	

Total

In words	
In numbers	

Code Numbers

Marking Examiner	
Checked by	
Superviced by	

Part A

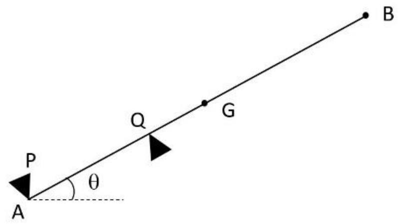
1. Two particles A and B of masses in the ratio $n:1$, moving on a smooth horizontal table along a same straight line towards each other with speed u and $2u$ respectively, collide directly. If the coefficient of restitution between A and B is $\frac{1}{3}$, find the velocity of B, just after the collision, in terms of u and n.
   If the particle B reverse just after the collision with velocity u, deduce that $n = 3$.

2. A particle is projected with velocity u, at an angle $\alpha \left(0 < \alpha < \frac{\pi}{2}\right)$ to horizontal with respect to the OXY plane, when it passes through a point (x, y). It is given that $y = x \tan\alpha - \frac{gx^2 \sec^2\alpha}{2u^2}$.
   When $\theta = 30^0$, if R is the horizontal range of the particle, show that the vertical height of the particle is $\frac{\sqrt{3}R}{16}$, when it has moved a horizontal distance of $\frac{3R}{4}$.

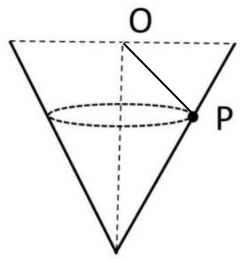
2. In a motor race, two cars A and B starts their motions with velocities $2u$ and u respectively ,on the horizontal track and maintain uniform accelerations f and $2f$ respectively.
Draw the velocity-time graph for the motion of car A relative to car B, until their velocities are equal.
Hence show that the maximum distance between A and B is $\frac{u^2}{2f}$.

3. A vehicle of mass $M\ kg$ with its engine working at power $H\ kW$, has a maximum speed $u\ ms^{-1}$ on a horizontal road. Find the total resistance to the motion. When the vehicle is travelling with a constant speed $\frac{u}{2}\ ms^{-1}$ up a track of inclination 1: 60, the engine is working with 80% of it's original power and the total resistance has increased by 50% . Show that $H = \frac{uMg}{6000}$

4. AB is a uniform rod of length $2a$. G is the center of gravity of AB. Its end A is in contact with a smooth peg P, and fall over another rough peg Q as shown in the figure. The road is in limiting equilibrium, inclining θ to horizontal, and the coefficient of friction between the road and the peg Q is $\frac{1}{4}\ tan\ \theta$. Show that the length PQ is $\frac{a}{4}$.



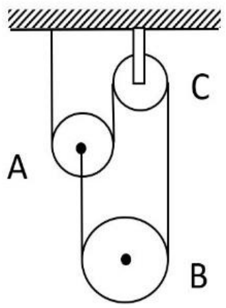
5. A hollow cone of semi vertical angle α is fixed vertically, with its vertex downward as shown in the figure. A particle P of mass m has placed on the smooth inner surface of the cone and connected to the center O, by an inelastic string of length a. This string makes an angle α with the downward vertical. When this particle is subjected to an angular velocity ω, show that the tension of the string is $T = \frac{m[\ g.cos\alpha - a\omega^2 sin^2\alpha]}{cos2\alpha}$



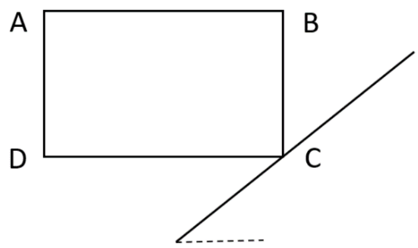
6. In the usual notation, with respect to a fixed origin O, let the position vectors of three points A, B and C are $4\,i + j,\ \lambda\, i + \mu j$ *and* $i + 5j$ respectively, where $\lambda, \mu \in \mathbb{R}^{+}$. If the diagonals of the quadrilateral OABC are equal in length and perpendicular to each other, show that $\lambda = 4$ and $\mu = 3$.

7. The figure shows a system of pulleys. A and B are two smooth movable pulleys of masses m and M respectively. C is a smooth pulley fixed to a ceiling. One end of a light inextensible string is fixed to a point on the celling and passes over the pulleys and the other end of it is fixed to the pulley A, as shown in the figure. The portions of the strings not in contact with the pulleys are vertical.

When the system is released from rest, write sufficient equations to determine the tension of the string.



9. A uniform rectangular lamina ABCD of weight $64\,N$ has side $AB = 120\,cm$ and $AD = 32\,cm$, is in equilibrium, in a vertical plane with corner A is freely hinged to a fixed point and the corner C is in contact with a smooth plane inclined at angle $tan^{-1}\frac{15}{8}$ to the horizontal, and the side AB is horizontal.



Find the magnitude and direction of the hinge reaction on the lamina.

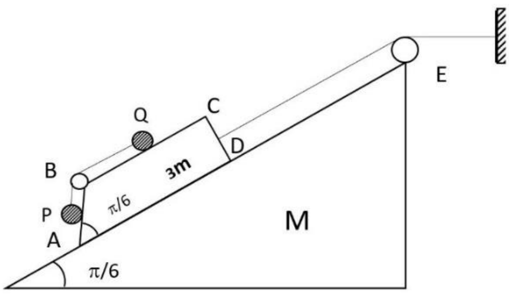
10. Let A and B are two exhaustive events defined on the sample space Ω. Given that $P(A^{/}\cap B) = 0.1$ and $P(B^{/}\cap A) = 0.6$, where $A^{/}$and $B^{/}$ denote the complement of A and B respectively.
Find are $P(A\cap B)$. Are the events A and B independent? Justify your answer.

Part B

11. (a) A small stone P is projected vertically upward with a velocity u from a point O. When it has travelled half of it's maximum height, the stone has divided into two similar parts A and B. Instantaneously if A become rest, then show that the initial velocity of B is $\sqrt{2}\,u$.
Draw the velocity-time graph for the motion of the particles A and B in a same diagram.
Using the graph, show that the velocities of A and B are equal, when the particle A reaches the point of projection O.
Also show that the vertical displacement of the particle B, from the point of projection is $\frac{u^2}{g}$, at this moment.

(b) Two places A and B are at the same height above the ground level , which are at a horizontal distance $a\ km$. X and Y are two aircrafts placed at A and B respectively. Both aircrafts can fly with a constant velocity $\boldsymbol{\lambda u}\ kmh^{-1}(\lambda > 1)$ in still air. When a wind blows uniformly with a speed of $\boldsymbol{u}\ kmh^{-1}$from a direction making an angle $\frac{\pi}{3}$ with the line of AB ,the aircrafts X and Y start to fly simultaneously so as to complete the path from A to B and B to A ,respectively .
Draw velocity triangles for the motion of both aircrafts X and Y in a same diagram.
Hence show that the aircrafts will collied after a time $\frac{a}{u\sqrt{4\lambda^2-3}}$ hours.

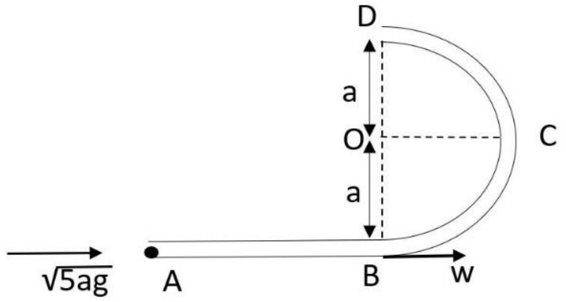
12. (a) ABCD is the vertical cross section through the center of gravity of a smooth uniform block of mass $3m$.
The face containing AD is placed on a smooth face inclined $\frac{\pi}{6}$ to horizontal, of a moveable wedge of mass M as shown in the figure.
The wedge is free to move on a smooth horizontal floor.
AB line is the greatest slope of the face of ABCD plane with BC and AD are parallel faces.



Two similar particles P and Q, each of mass m are connected by an inelastic light string, which passing over a light smooth pulley fixed to the block at B. One end of another light inextensible string connected to the line CD, passing over another light smooth pulley fixed to the block at E, and its other end attached to a vertical wall perpendicularly. Strings are taught in the position shown in the diagram and the system is released from the rest from this position.
Obtain sufficient equations to determine the acceleration of each particle, block and the wedge.

(b) A thin tube ABCD is fixed in a vertical plane with AB horizontal as shown in the figure. BCD is a smooth semicircular part of center O and radius a, while AB is a rough thin tube of length a.

A particle of mass $2m$ is placed inside the tube at the point A, and it is given a velocity of magnitude $\sqrt{5ga}$ in the direction $\overline{AB}$. The coefficient of friction between the particle and the tube AB is $\frac{1}{2}$.



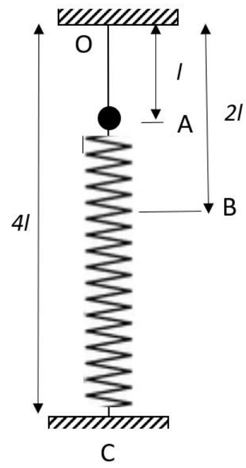
If the velocity of the particle at the point B is w, show that $w = 2\sqrt{ag}$.

Then the particle moves through the circular tube.
When the particle turns through an angle θ with the downward vertically, $(0 < \theta < \pi)$, show that the speed v of the particle at this instant is given by $v^2 = 2ag(\cos\theta + 1)$.

If the reaction on the particle from the tube at this instant is R, show that $R = 2mg(2 + 3\cos\theta)$.

Hence find the angle of which the particle reverse the direction of its contact reaction inside the tube. Deduce that the particle could be able to just reach the highest point D.

13. One end of a light elastic string of natural length l and the modulus of elasticity mg is attached to the fixed point O and the other and is attached to a particle P of mass m.
One end of a light spring of natural length $2l$ and the modulus of elasticity mg is attached to a fixed point C, which is at distance $4l$, vertically below to O, and the other end of the string is attached to the particle P, as shown in the figure.



Now the particle P is raised to the point A, which is at a distance l from O, and gently released from rest. B is a point on OC such that $OB = 2l$.
Let $OP = x$, where $x > l$, Show that the motion of the particle from A to B is given by $\ddot{x} + \frac{3g}{2l}(x - 2l) = 0$

Rewrite this equation in the form of $\ddot{X} + \omega^2 X = 0$ where $X = x - 2l$ and ω is a constant to be determine. Find the center of this motion.

Using the formula $\dot{X}^2 = \omega^2(c^2 - X^2)$, find the amplitude c and show that the velocity of the particle P, when it reaches B is $\sqrt{\frac{3gl}{2}}$.

Let D is the lowest point reached by the particle P.
Show that the motion of the particle P, from B to D is given by $\ddot{x} + \frac{g}{2l}(x - 2l) = 0$

Rewrite the equation in the form of $\ddot{Y} + \omega_0^2 Y = 0$ where $Y = x - 2l$ and ω_0 is a constant to be determined. Find the center of this motion.

Using the formula $\dot{Y}^2 = \omega_0^2(k^2 - Y^2)$, find the amplitude k, of this motion.

Show that the vertical distance from O to the lowest point D reached by the particle P is $(\sqrt{3} + 2)l$.

Also show that the total time for the motion of the particle from A to D is $\pi\sqrt{\frac{l}{6g}}(1 + \sqrt{3})$.

14. (a) Let $\underline{a}$ and $\underline{b}$ are non zero non parallel vectors and α , $\beta \in \mathbb{R}$.
If $\alpha\underline{a} + \beta\underline{b} = \underline{0}$, then Show that $\alpha = 0$ and $\beta = 0$.
O, A and B are non colinear three points with position vectors $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$ and $\overrightarrow{OC} = \underline{a} + 2\underline{b}$.
D is a point on BC such that BD : DC = 1 : 2.

Find the position vector $\overrightarrow{OD}$ in terms of $\underline{a}$ and $\underline{b}$.
The line AB and OD meet at E. Explain why $\overrightarrow{AE} = \lambda\,\overrightarrow{AB}$ and $\overrightarrow{OE} = \mu\,\overrightarrow{OB}$ where $\lambda, \mu \in \mathbb{R}$.
Using vector triangle addition, write $\overrightarrow{OE}$ in terms of $\underline{a}$ and $\underline{b}$. Hence show that $\overrightarrow{OE} = \frac{1}{5}(\underline{a} + 4\underline{b})$.

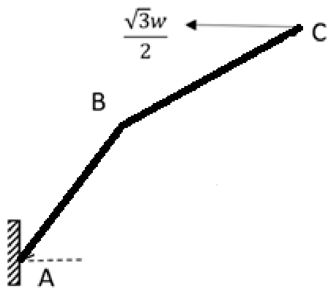
(b) Let ABC be an equilateral triangle of side a with AB horizontal. D, E and F are the mid points of AB, BC and AC respectively. Forces of magnitudes λP, μP, 8P, $4\sqrt{3}$P, $6\sqrt{3}$P and $10\sqrt{3}$P acts along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{BF}$, $\overrightarrow{EA}$ and $\overrightarrow{CD}$ respectively, where λ and μ are real constants.

(i). Show that this system could not be in equilibrium for any values of λ and μ.

(ii). If the system is reduced to a couple, find the value of λ and μ. Also find the magnitude and sense of this couple.

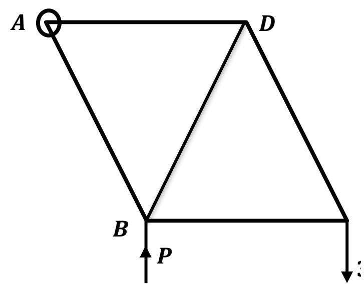
(iii). If the system is reduced to a single force of $28P$ and a clockwise moment of $2\sqrt{3}Pa$ about the vertex A, find the values of λ and μ.
Find the direction of this single force and the distance from A to the point at which the line of action of the resultant cuts the side AB.

15. (a) AB and BC are two rods, each of weight w and in equal length. They are smoothly jointed at B, and hinged to a vertical wall at A. The system is in equilibrium in a vertical plane by means of a horizontal force of $\frac{\sqrt{3}w}{2}$ applied at C, as shown in the figure.



Find the acute angle made by the rod BC with horizontal and the angle subtended by the rod AB, to the upward vertical.
Also find the magnitude and direction of the reaction at the joint B, on the rod BC.

(b) The framework shown in the adjoining figure is made from five light rods AB, BC, CD, AD and BD each of equal length freely joined at A, B, C and D. There is a load of $3w$ hanging at C.
The Framework is smoothly hinged at A to a fixed point and kept in equilibrium in a vertical plane with BC horizontal, by means of a vertical force p applied at B, as shown in the figure.



Find the value of p in terms of w.
Draw a stress diagram for the joints B,C and D, using Bow's notation and hence find stress in each rod, stating whether they are tensions or trusts.

16.

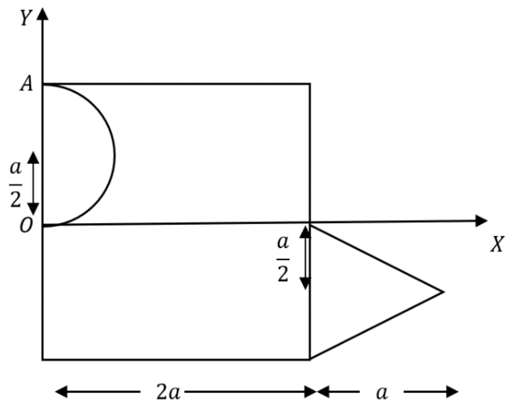
Show that the center of mass of

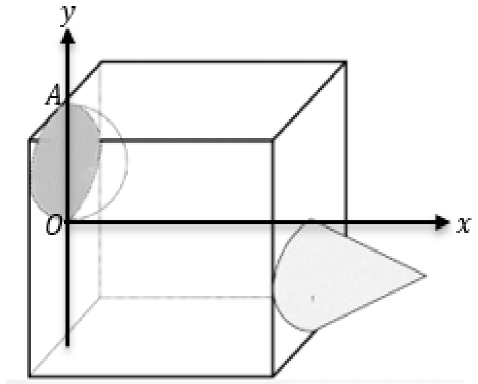
(i) a uniform solid right circular cone of base radius r and height h is at a distance $\frac{h}{4}$ from the center of the base.

(ii) a uniform solid hemisphere of radius r is at a distance $\frac{3r}{8}$ from its center.

A wooden block is in the shape of a cuboid of length $2a$ width a and height $2a$. A solid hemisphere of radius $\frac{a}{2}$, has carved out form the upper part of a side of the block.

A right circular cone of radius $\frac{a}{2}$ and height a is attached to the lower part of the opposite side of the block ,as shown in the figure. The density of the cone is doubled as the density of the block.





By getting the OXY axes through the midpoints through opposite sides and perpendicular to it, as shown in the figure, show that the center of gravity of the composite body lies $\frac{3a\pi}{2(48+\pi)}$ distance below OX axis and $\frac{3(256+23\pi)}{16(48+\pi)}a$ distance from $OY axis$.

Now the composite body is freely hinged at A. Find the inclination of OA to the vertical.

A horizontal force P is applied at the vertex of the cone such that the OX axis to be horizontal.

If the weight of the composite body is w, show that $P = \frac{(256+23\pi)}{8(48+\pi)}w$.